Challenging Partial, Approximate and Partial Approximate Measurement Invariance

Artur Pokropek, Polish Academy of Science and EC Joint Research Centre
Peter Schmidt, University of Giessen
Eldad Davidov, University of Cologne and University of Zurich

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Outline

• Background (empirical approaches for testing comparability)
• Aim of the study
• Design of simulations
• Results of simulation study
• Conclusions
• Limitations and further work
Small „natural” differences among groups

<table>
<thead>
<tr>
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Small „natural” differences among groups

<table>
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<td>Non invariance</td>
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<td>Metric invariance (weak invariance)</td>
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<tr>
<td>Scalar invariance (strong invariance)</td>
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<tr>
<td>Configural invariance</td>
<td></td>
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<tr>
<td>Full invariance</td>
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Multi-Group CFA
Small „natural” differences among groups

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<table>
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<th>Items</th>
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<td>Full invariance</td>
<td>Multi-Group CFA</td>
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NO

Yes
Partial invariance

Small „natural” differences among groups

Large differences among some groups and some items

Full invariance
Multi-Group CFA

Scalar invariance
(strong invariance)

Metric invariance
(weak invariance)

Configural invariance

Non invariance

NO

Yes

Approximate invariance
BSEM

0.001

0.01

0.1
Small „natural” differences among groups

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<tr>
<td><strong>Approximate partial invariance</strong> Alignment, Partial BSEM</td>
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Large differences among some groups and some items
Two general questions

• **Under what conditions deviations from strict invariance allow for conducting meaningful comparisons of model parameters (latent means and unstandardized regression coefficients across groups)**
  – Scalar, metric, configural invariance
  – Partial invariance
  – Approximate invariance
  – Partial approximate invariance

• **How good are the tools for detecting different types of measurement invariance**
  – Scalar, metric, configural invariance
  – Partial invariance
  – Approximate invariance
  – Partial approximate invariance
Two general questions

• Under what conditions deviations from strict invariance allow for conducting meaningful comparisons of model parameters (latent means and unstandardized regression coefficients across groups)
  – Scalar, metric, configural invariance ← well studied
  – Partial invariance
  – Approximate invariance ← limited number of studies
  – Partial approximate invariance

• How good are the tools for detecting different types of measurement invariance
  – Scalar, metric, configural invariance ← well studied
  – Partial invariance
  – Approximate invariance ← some pieces are missing
  – Partial approximate invariance
Aim of the study

• Under what conditions deviations from strict invariance allow for conducting meaningful comparisons of model parameters (latent means and unstandardized regression coefficients across groups)
  – scalar, metric, configural invariance ← well studied
  – Partial invariance
  – Approximate invariance ← limited number of studies
  – Partial approximate invariance

• How good are the tools for detecting different types of measurement invariance
  – scalar, metric, configural invariance ← well studied
  – Partial invariance invariance
  – Approximate invariance ← some pieces are missing
  – Partial approximate invariance
Design of simulations

True Model
Design of simulations

True Model
Design of simulations

\[ F_1, \lambda_1, \tau_1 \]
\[ F_2, \lambda_2, \tau_2 \]
\[ F_3, \lambda_3, \tau_3 \]
\[ F_n, \lambda_n, \tau_n \]

True Model
Design of simulations

\[ \lambda_1, \tau_1 \]
\[ \lambda_2, \tau_2 \]
\[ \lambda_3, \tau_3 \]
\[ \lambda_4, \tau_4 \]
\[ \lambda_5, \tau_5 \]
\[ \lambda_1, \tau_1 \]
\[ \lambda_2, \tau_2 \]
\[ \lambda_3, \tau_3 \]
\[ \lambda_4, \tau_4 \]
\[ \lambda_5, \tau_5 \]
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\[ \lambda_2, \tau_2 \]
\[ \lambda_3, \tau_3 \]
\[ \lambda_4, \tau_4 \]
\[ \lambda_5, \tau_5 \]

Invariance holds

\[ \lambda_a, \tau_a + \text{PMI bias} \]
\[ \lambda_2, \tau_2 + \text{AMI bias} \]
\[ \lambda_3, \tau_3 + \text{AMI bias} \]
\[ \lambda_4, \tau_4 + \text{AMI bias} \]
\[ \lambda_5, \tau_5 + \text{AMI bias} \]
\[ \lambda_1, \tau_1 + \text{AMI bias} \]
\[ \lambda_b, \tau_b + \text{PMI bias} \]
\[ \lambda_3, \tau_3 + \text{AMI bias} \]
\[ \lambda_4, \tau_4 + \text{AMI bias} \]
\[ \lambda_5, \tau_5 + \text{AMI bias} \]
\[ \lambda_1, \tau_1 + \text{AMI bias} \]
\[ \lambda_2, \tau_2 + \text{AMI bias} \]
\[ \lambda_3, \tau_3 + \text{AMI bias} \]
\[ \lambda_4, \tau_4 + \text{AMI bias} \]
\[ \lambda_5, \tau_5 + \text{AMI bias} \]

True Model

Non-invariance

(partial or/and aprox.)
Design of simulations

Invariance holds

Non-invariance (partial or/and aprox.)

1. **tMG-CFA** - Multigroup CFA model using data without non-invariance

2. **MG-CFA** - Multigroup CFA scalar mode that ignores MI

3. **PMG-CFA** - Partial invariance multi-group confirmatory factor analysis (Byrne et al., 1989).

4. **MG-BSEM** - Multi-group Bayesian SEM

5. **PMG-BSEM** - Partial invariance Multi-group Bayesian SEM (Muthén & Asparouhov, 2013).

6. **AMG-CFA** - MG-CFA with alignment optimization (fixed) (Asparouhov & Muthén, 2014; Muthen & Asparouhov, 2014)
Design of simulations (details I)

1. We sampled means and standard deviations for each group from normal distributions $N(0, 0.3)$ and $N(1, 0.1)$ respectively.

2. Three random variables were generated for each group: $F$ - representing a latent trait and two variables representing two criterion variables ($C_1$ and $C_2$). The $C$ variables were generated from standard normal distributions in such a way so that the regression coefficient $C_1 \leftarrow F$ was set to $0.3$ in each group while $C_2 \leftarrow F$ was set to $0.1$ in each group.

3. We generated parameters for each item.
   a. Factor loadings from a uniform distribution $[0.5, 0.8]$
   b. Intercepts from a uniform distribution $[-0.15 \text{ and } 0.15]$

4. Factor indicators were randomly generated according to the sampled item parameters.

5. The continuous factor indicators were discretized into five categories while using for the thresholds the values $-1.30, -0.47, +0.47, +1.30$ (a similar approach picking up these thresholds was used in Sass, Schmitt, & Marsh (2014)).
Design of simulations (details II)

6. For our simulation settings these parameters resulted in scales with relatively high reliabilities (Cronbach's alpha):
   - 0.75 for the 3-item scale,
   - 0.80 for the 4-item scale,
   - 0.85 for the 5-item scale.

7. We added a non-invariance bias in specific affected groups.

8. Items were independently sampled from each group. In each replication, random assignment of non-invariance was repeated so that no particular pattern of non-invariance was present (in each replication different items were sampled to be non-invariant).

9. Bias was added to the selected items. We added a bias of +0.2 or -0.2 both for factor loadings and intercepts. The sign of the bias was determined randomly, independently for each item and for each item parameter.

10. Approximate MI was added to rest of the items if it was considered in the condition (0.000, 0.001, 0.005, 0.010, 0.025, 0.050)
Simulation conditions for this study

- Number of Groups: **24**
- Sample Size: **1500**
- Number of items per scales: **3, 4, 5**
- Number of non-invariant items per group (full non-invariance): **1,2,3,4**
- Groups affected by fully non-invariant items: **25%, 50%, 75%, 100%**
- Approximate MI, prior dif. variance: **0.000, 0.001, 0.005, 0.010, 0.050**
- **400** replications
Simulation conditions for this study

- Number of Groups: 24
- Sample Size: 1500
- Number of items per scales: 3, 4, 5
- Number of non-invariant items per group (full non-invariance): 1, 2, 3, 4
- Groups affected by fully non-invariant items: 25%, 50%, 75%, 100%
- Approximate MI, prior dif. variance: 0.000, 0.001, 0.005, 0.010, 0.050
- 400 replications

Ranking recovery and/or precise means recovery practically impossible (regression path coefficients OK)
Simulation conditions for this study

• Number of Groups: **24**
• Sample Size: **1500**
• Number of items per scales: **5**
• Number of non-invariant items per group (full non-invariance): **1,2,3,4**
• Groups affected by fully non-invariant items: **25%, 50%, 75%, 100%**
• Approximate MI, prior dif. variance: **0.000, 0.001, 0.005, 0.010, 0.050**
• **400** replications
Evaluation criteria

• **Means Correlations** - according to Muthén and Asparouhov’s (2014) and Muthén and Asparouhov’s (2013) recommendation, a correlation between the true value of the means and their estimates of at least \(0.98\) (and preferably \(0.99\)) indicates a reasonably good recovery of the mean rankings.

• **95% CI Coverage** - the coverage of the true means by the 95% coefficient intervals (CI) generated using standard errors of the estimated means. (90%-100%)

• **Average estimates** - For estimates of path coefficients, we are looking at average estimate over the groups and replications for bias detection (0.29-0.31)

• **RMSE (Root Mean Squared Error)** – for overall accuracy (<0.06)
Small „natural” differences among groups

- Non invariance
- Metric invariance (weak invariance)
- Configural invariance
- Scalar invariance (strong invariance)
- Full invariance
- Multi-Group CFA

Yes

Approximate invariance
BSEM

NO

Partial invariance
Multi-Group CFA

Large differences among some groups and some items

Approximate partial invariance
Alignment, BSEM Alignment, Partial BSEM

0.001
0.01
0.1

0.001
0.01
0.1
## Recovery of group means for partial MI situation

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<thead>
<tr>
<th>Groups</th>
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<th>MG-CFA</th>
<th>PMG-CFA</th>
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<th>PMG-BSEM</th>
<th>AMG-CFA</th>
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<td>RMSE</td>
<td>95C</td>
<td>Cor</td>
<td>RMSE</td>
<td>95C</td>
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<td>0.040</td>
<td>0.948</td>
<td>0.989</td>
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<td>0.948</td>
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<tr>
<td>50%</td>
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<td>0.994</td>
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<td>0.948</td>
<td>0.985</td>
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## Recovery of path coefficients for for partial MI situation

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<th>Non-invariant</th>
<th>MG-CFA (no MI)</th>
<th>MG-CFA</th>
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<td>mean RMSE 95C</td>
<td>mean RMSE 95C</td>
<td>mean RMSE 95C</td>
<td>mean RMSE 95C</td>
<td>mean RMSE 95C</td>
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<td>25%</td>
<td>1  0.303 0.007 0.915</td>
<td>0.302 0.007 0.909</td>
<td>0.302 0.007 0.917</td>
<td>0.288 0.014 0.894</td>
<td>0.298 0.010 0.933</td>
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<td>2  0.302 0.007 0.918</td>
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<td>0.280 0.020 0.845</td>
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Small “natural” differences among groups

- Full invariance
  - Multi-Group CFA

- Partial invariance
  - Multi-Group CFA

- Approximate invariance
  - BSEM

- Approximate partial invariance
  - Alignment, BSEM Alignment, Partial BSEM

- Scalar invariance (strong invariance)
- Metric invariance (weak invariance)
- Configural invariance
- Non invariance
Recovery of group means for AMI situation

<table>
<thead>
<tr>
<th>AMI</th>
<th>MG-CFA</th>
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<th>MG-BSEM</th>
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<th>AMG-CFA</th>
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<tbody>
<tr>
<td></td>
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Recovery of path coefficients for AMI situation

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How good are the tools for detecting different type of measurement invariance

1. Posterior Predictive P-values (PPP),
2. Bayesian Information Criterion (BIC),
3. Deviance Information Criterion (DIC).
How good are the tools for detecting different type of measurement invariance

<table>
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<tr>
<th>Priors for differences in MG-BSEM</th>
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<th>Approximate measurement (non)invariance</th>
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<tr>
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<td>DIC</td>
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</table>
How good are the tools for detecting different type of measurement invariance

<table>
<thead>
<tr>
<th>MG-BSEM</th>
<th>Fit measure</th>
<th>Approximate measurement (non)invariance</th>
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<td></td>
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<td>DIC</td>
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<table>
<thead>
<tr>
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<th>Fit measure</th>
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<td>0.000</td>
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<td>0.176  0.202  0.010  0.000  0.000  0.000</td>
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<td>DIC</td>
<td>0.000  0.000  0.020  0.189  0.772  0.977</td>
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Small "natural" differences among groups

- No large differences among some groups and some items
  - Non invariance
  - Configural invariance
  - Metric invariance (weak invariance)
  - Scalar invariance (strong invariance)
  - Full invariance
  - Multi-Group CFA

- Yes large differences among some groups and some items
  - Partial invariance
  - Multi-Group CFA

- Yes approximate partial invariance
  - Alignment, BSEM Alignment, Partial BSEM

- Yes approximate invariance
  - BSEM

- Yes full invariance
  - Multi-Group CFA

- No small "natural" differences among groups
  - Partial invariance
  - Multi-Group CFA

- Yes scalar invariance (strong invariance)
  - BSEM

- No scalar invariance (strong invariance)
  - BSEM
## Recovery of group means for partial + AMI situation

<table>
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<th>MG-CFA</th>
<th>PMG-CFA</th>
<th>MG-BSEM</th>
<th>PMG-BSEM</th>
<th>AMG-CFA</th>
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<tr>
<td></td>
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## Recovery of group means for partial + AMI

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<th>Groups affect.</th>
<th>AMI</th>
<th>Non-invariant</th>
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<th>PMG-CFA</th>
<th>MG-BSEM</th>
<th>PMG-BSEM</th>
<th>AMG-CFA</th>
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<tbody>
<tr>
<td></td>
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<td>RMSE</td>
<td>95C</td>
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<td>25%</td>
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<td>1</td>
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<td>0.010</td>
<td>0.884</td>
<td>0.304</td>
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Conclusions

• Don’t use short scales (3-, 4-items)
• PI situation is not a problem for PMG (if non-invariance items are known)
• PI situation is not a problem for AMG models (if there is little non-invariance item)
• Approximate MI (AMI) is problematic
  – AMI>0.001 recovery of means is very difficult
  – SEM Path coefficients are reasonably robust (even MG-CFA will do)
• AMI with partial non-invariance is even more problematic
Limitations and further work

• Add additional conditions which are useful and realistic for applied researchers
• Problem of detection of biased items in partial invariance situations
• Treat Partial Invariance (PI) items as unknown in similar simulations
  – Assess effects of misspecification of PI items
• Impact of MI on more complicated SEM models